$\boxed{\text{Sequences + Series WS: Solutions}}$

1.) $a_9 = -5 + 4(9-1)$
$\boxed{a_9 = 27}$

2.) $a_n = a_1(r)^{n-1}$
$a_4 = 5(3)^{4-1} = \boxed{135}$

3.) $a_6 = 3 + -4(6-1)$
$a_6 = 3 + -4(5) = -17$

4.) $a_6 = -4(-2)^{6-1} = -4(-2)^5$
$= -4(-32)$
$= \boxed{128}$

5.) $a_6 = a_1 \cdot r^{6-1} \rightarrow 54 = a_1 r^5$
$a_3 = a_1 \cdot r^{3-1} \rightarrow \div\ 2 = a_1 r^2$
$27 = r^3 \rightarrow \sqrt[3]{27} = r \rightarrow \underline{3 = r}$

* plug $r = 3$ into either equ. + solve for a_1
$2 = a_1(3)^2 \rightarrow 2 = a_1 \cdot 9 \rightarrow \frac{2}{9} = a_1$

* keep mult. by r to get the remaining values

$\underline{\frac{2}{9}}$, $\underline{\frac{2}{3}}$, 2, $\underline{6}$, $\underline{18}$, 54

6.) $a_4 = a_1 + d(4-1) \rightarrow 20 = 3 + 3d \rightarrow \frac{17}{3} = d$
$a_2 = 3 + \frac{17}{3} = \frac{26}{3}$; $a_3 = \frac{26}{3} + \frac{17}{3} = \frac{43}{3}$

7.) $a_4 = a_1 + d(4-1) \rightarrow 27 = 5 + 3d \rightarrow \frac{22}{3} = d$
$a_2 = 5 + \frac{22}{3} = \frac{37}{3}$; $a_3 = \frac{37}{3} + \frac{22}{3} = \frac{59}{3}$

8.) $162 = 32 \cdot r^4$ $[a_5 = a_1(r)^{5-1}] \rightarrow \frac{162}{32} = \frac{\cancel{32} r^4}{\cancel{32}}$
$\sqrt[4]{\frac{81}{16}} = r \rightarrow \frac{3}{2} = r$ $a_2 = 32\left(\frac{3}{2}\right)^1 = \underline{48}$, $a_3 = 32\left(\frac{3}{2}\right)^2 = \underline{72}$
$a_4 = 32\left(\frac{3}{2}\right)^3 = \underline{108}$

19.) $-120 = \frac{16(a_1+15)}{2} \rightarrow -240 = 16a_1 + 240 \rightarrow \frac{-480}{16} = \frac{16a_1}{16}$

$-30 = a_1$

$15 = -30 + d(16-1)$

$3 = d$

1^{st} 3 terms

$-30, -27, -24$

9.) $a_6 = a_1 + d(6-1) \rightarrow 14 = a_1 + 5d$

$a_2 = a_1 + d(2-1) \rightarrow (-)\ -10 = a_1 + 1d$

$24 = 4d \rightarrow 6 = d$

$14 = a_1 + 5(6) \rightarrow -16 = a_1$ $\quad \underline{-16}, -10, \underline{-4}, \underline{2}, \underline{8}, 14$

10.) $a_{15} = a_1 + d(15-1) \rightarrow a_{15} = -3 + 6(14) = \boxed{81}$

11.) $a_n = (-6)(-2/3)^{n-1} \rightarrow a_1 = -6;\ a_2 = (-6)(\frac{-2}{3})^1 = 4$

$a_3 = (-6)(\frac{-2}{3})^2 = \frac{-8}{3};\ a_4 = \frac{16}{9}$

12.) $a_6 = 160(\frac{1}{2})^5 = \underline{5}$; $S_6 = 160\left[\frac{1-(\frac{1}{2})^6}{1-1/2}\right] = \boxed{315}$

13.) $a_7 = 5(-\frac{1}{2})^6 = \frac{5}{64}$; $S_7 = 5\left[\frac{1-(-\frac{1}{2})^7}{1-\frac{1}{2}}\right] = \boxed{\frac{645}{64}}$

14.) $a_{21} = 13 + -6(21-1) \rightarrow a_{21} = -107$

$S_{21} = \frac{21(13+-107)}{2} = \boxed{-987}$

15.) $a_{16} = 44 \rightarrow 44 = a_1 + -\frac{2}{3}(16-1) \rightarrow 44 = a_1 + -10$

$54 = a_1$

$S_n = \frac{16(54+44)}{2} = \boxed{784}$

16.) $-55 = a_1\left[\frac{1-(\frac{-2}{3})^5}{1-(\frac{-2}{3})}\right] \rightarrow -81 = a_1$

1^{st} 3 terms

$\boxed{14, 11, 8}$

$\frac{-2414}{-71} = \frac{-71n}{-71}$

17.) $a = 3072$

18.) $-85 = 14 + d(n-1)$

$-1207 = \frac{n(14+-85)}{2}$

$34 = n$

$-85 = 14 + d(34-1) \rightarrow d = -3$

SEQUENCES AND SERIES WORKSHEET

NAME: Solutions

Use the formulas provided to you to complete the following. Determine what type of sequence the following are and then complete the problem.

1. $a=-5$, $d=4$, $n=9$; find the n^{th} term

   $a_9 = 27$

2. $a=5$, $n=4$, $r=3$; find the n^{th} term

   $a_4 = 135$

3. $a=3$, $d=-4$, $n=6$; find the n^{th} term

   $a_6 = -17$

4. $a=-4$, $n=6$, $r=-2$; find the n^{th} term

   $a_6 = 128$

Find the missing terms in each sequence. You are given what type of sequence represents each one.

5. 2/9, 2/3, 2, 6, 18, 54 (geometric)

6. 3, $\frac{26}{3}$, $\frac{43}{3}$, 20 (arithmetic)

7. 5, $\frac{37}{3}$, $\frac{59}{3}$, 27 (arithmetic)

8. 32, 48, 72, 108, 162 (geometric)

9. -16, -10, -4, 2, 8, 14 (arithmetic)

10. Find the 15^{th} term for the arithmetic sequence -3, 3, 9, ...

    81

11. Find the first 4 terms of the geometric sequence with $a=-6$ and $r=-2/3$

    $-6, 4, -8/3, 16/9$

Find S_n for each series described. You will need to determine if the series is arithmetic or geometric.

12. $160 + 80 + 40 + \ldots$, $n=6$

    315

13. $a=5$, $r=-1/2$, $n=7$

    $\frac{645}{64}$

14. $a=13$, $d=-6$, $n=21$

    -987

15. $d=-2/3$, $n=16$, $u_n=44$

    784

Find "a" for each geometric series.

16. $S_n=-55$, $r=-2/3$, $n=5$

    -81

17. $S_n=2457$, $\boxed{a=3072}$, $r=-4$

    3072

Find the first 3 terms of each arithmetic series.

18. $a=14$, $u_n=-85$, $S_n=-1207$

    14, 11, 8

19. $n=16$, $u_n=15$, $S_n=-120$

    $-30, -27, -24$

Infinite Geometric Sequences

On each bounce, Ball 1 reaches a height equal to 3/4 of the height of its previous bounce. On the first bounce, it achieves a height of 25 feet.

Ball 2, which reaches a height of 18 feet on its first bounce, bounces 4/5 of the height of its previous bounce on each bounce.

1. Fill in the table, showing the height of each ball for each bounce.

Bounce	Ball 1 height (ft.)	Ball 2 height (ft.)
1	25	18
2	18.75	14.4
3	14.063	11.52
4	10.547	9.216
5	7.9102	7.3728
6	5.9326	5.8982
7	4.4495	4.7186

2. Will Ball 2 ever bounce higher than Ball 1? If so, at which bounce?

yes, 7th

3. For how many bounces do both balls bounce above 10 feet?

3

4. When do the balls "stop" bouncing (i.e., achieve a height of less than 3 inches)?

Ball 1 → 9th bounce ; Ball 2 → 10th bounce

5. What minimum initial bounce height (to the nearest foot) would you have to ensure for each ball in order to guarantee that it bounces at least 8 feet high by the sixth bounce?

$8 = a_1\left(\frac{3}{4}\right)^{6-1} \rightarrow 8 = a_1\left(3/4\right)^5 \rightarrow$ 34 Ft [Ball 1]

$8 = a_1\left(4/5\right)^{6-1} \rightarrow 8 = a_1\left(4/5\right)^5 \rightarrow$ 24 Ft [Ball 2]

6. Calculate the sum of all of the bounce heights for both balls.

Ball 1 $\rightarrow S_8 = 25\left[\frac{1-(3/4)^8}{1-(3/4)}\right] = 89.9887$; Ball 2 $\rightarrow S_9 = 18\left[\frac{1-(4/5)^9}{1-4/5}\right] = 77.9204$

$89.9887 + 77.9204 = \boxed{167.9091 \text{ Ft}}$

Follow-up

7. What is it about the fractions involved in this problem that makes the ball react in the manner described?

• The larger the Fraction, the more significant the bounce height.